Reference: Thurstone, L. L. (1934). The vectors of the mind. *Psychological Review*, *41*, 1-32.

The Vectors of Mind

L. L. Thurstone

Under the title of this address, "The Vectors of Mind," I shall discuss one of the oldest of psychological problems with the aid of some new analytical methods. I am referring to the old problem of classifying the temperaments and personality types and the more recent problem of isolating the different mental abilities.

Until very recently the only attempt to solve this problem in a quantitative way seems to have been the work of Professor Spearman and his students. Spearman has formulated methods for dealing with the simplest case, in which all of the variables that enter into a particular study can be regarded as having only one factor in common. The factor theory that I shall describe starts without this limitation, in that I shall make no restriction as to the number of factors that are involved in any particular problem. The resulting factor theorems are quite different in form and in their underlying assumptions, but it is of interest to discover that they are consistent with Spearman's factor theory, which turns out to be a special case of the present general factor theory.

In this paper I shall first review the single-factor theory of Spearman. Then I shall describe a general factor theory. Those who have only a casual interest in the theoretical aspects of this problem will be more interested perhaps in the applications of the new factor theory to a number of psychological problems. These psychological applications will constitute the major part of this paper.

It is thirty years ago that Spearman introduced his single-factor method and the hypothesis that intelligence is a central and general factor among the mental abilities. The literature on this subject of factor analysis has tended temporarily to obscure his contribution, because the controversies about it have frequently been staged about rather trivial or even irrelevant matters. Professor Spearman deserves much credit for initiating the factor problem and for his significant contribution toward its solution, even though his formulation is inadequate for the multidimensionality of the mental abilities.

Spearman's theory has been called a two-factor method or theory. The two factors involved in it are, first, a general factor common to all of the tests or variables, and second, a factor that is specific for each test or variable. It is less ambiguous to refer to this method as a single-factor method, because it deals with only one common or general factor. If there are five tests with a single common factor and a specific for each test, then the method involves the assumption of one common and five specific factors, or six factors in all. We shall refer to his method less ambiguously as a single-factor method.

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We must distinguish between Spearman's method of analyzing the intercorrelations of a set of variables for a single common factor and his theory that intelligence is such a common factor which he calls "*g*". If we start with a given table of intercorrelations it is possible by Spearman's method, and also by other methods, to investigate whether the given coefficients *can* be described in terms of a single common factor plus specifics and chance errors. If the answer is in the affirmative, then we *can* describe the correlations as the effect of (1) a common factor, (2) a factor specific to each test, and (3) chance errors. In factor theory, the last two are combined because they are both unique to each test. Hence the analysis yields a summation of a common factor and a factor unique to each test. About this aspect of the single-factor method there should be no debate, because it is straight and simple logic.

But there can be debate as to whether we should describe the tests by a single factor even though one factor is sufficient. It is in a sense an epistemological issue. Even though a set of intercorrelations *can* be described in terms of a single factor, it is possible, if you like, to describe the same correlations in terms of two or three or ten or any number of factors.

The situation is analogous to a similar problem in physical science. If a particle moves, we designate the movement by an arrow-head, a vector, in the direction of motion, but if it suits our convenience we put two arrowheads or more so that the observed motion may be expressed in terms that we have already been thinking about, such as the *x, y,* and z axes. Whether an observed acceleration is to be described in terms of one force, or two forces, or three forces, that are parallel to the *x, y,* and *z* axes, is entirely a matter of convenience for us. In exactly the same manner we may postulate two or more factors in a correlation problem instead of one, even when one factor would be sufficient. To ask whether there "really" are several factors when one is sufficient, is as indeterminate as to ask how many accelerations there "really" are that cause a particle to move. If the situation is such that one factor is not adequate while two factors would be adequate, then we may think of two factors, but we may state the problem in terms of more than two factors if our habits or the immediate context makes that more convenient.

Spearman believes that intelligence can be thought of as a factor that is common to all the activities that are usually called intelligent. The best evidence for a conspicuous and central intellective factor is that if you make a list of stunts, as varied as you please, which all satisfy the common sense criterion that the subjects must be smart, clever, intelligent, to do the stunts well, and that good performance does not depend primarily upon muscular strength or skill or upon other non-intellectual powers, then the inter-stunt correlations will all be positive. It is quite difficult to find a pair of stunts, both of which call for what would be called intelligence, as judged by common sense, which have a negative correlation. This is really all that is necessary to prove that what is generally called intelligence can be regarded as a factor that is conspicuously common to a very wide variety of activities. Spearman's hypothesis, that it is some sort of energy, is not crucial to the hypothesis that it is a common factor in intellectual activities.

There is a frequently discussed difficulty about which more has been written than necessary. It has been customary to postulate a single common factor (Spearman's "*g*") and to make the additional but unnecessary assumption that there must be nothing else that is common to any pair of tests. Then the tetrad criterion is applied and it usually happens that a pair of tests in the battery has something else in common besides the most conspicuous single common factor. For example, two of the tests may have in common the ability to write fast, facility with geometrical figures, or a large vocabulary. Then the tetrad criterion is not satisfied and the conclusion is usually one of two kinds, depending on which side of the fence the investigator is on. If the investigator is out to prove "*g*," then he concludes that the tests are bad because it is supposed to be bad to have tests that measure more than one factor! If the investigator is out to disprove "*g*" then he shows that the tetrads do not vanish and that therefore there is no "*g*." Neither conclusion is correct. The correct conclusion is that more than one general factor must be postulated in order to account for the intercorrelations, and that one of these general factors may still be what we should call intelligence. But a technique for multiple factor analysis has not been available and consequently we have been stumbling around with "group factors" as the trouble-making factors have been called. A group factor is one that is common to two or more of the tests but not to all of them. I use the term common factor for all factors that extend to two or more of the variables. We see therefore that Spearman's criterion, limited as it is to a single common factor, is not adequate for proving or disproving his own hypothesis that there is a conspicuous factor that is common to all intelligence tests. If his criterion gives a negative answer it simply means that the correlations require more than one common factor. We do not need any factor methods at all to prove that a common factor of intelligence, is a legitimate postulate. It is proved by the fact that all intelligence tests are positively correlated.

There is only one limited problem for which Spearman's method is adequate, namely, the question whether a single factor is sufficient to account for the intercorrelations of a set of tests. The usual answer is negative. His criterion that the tetrads shall vanish is rarely satisfied in practice. One might wonder then why it is that numerous examples have been compiled in which the tetrad criterion is satisfied. The reason is simply this -- that in order to satisfy the criterion, the tests must be carefully selected so as to have only one thing in common. Another way by which the criterion is satisfied is to throw out of the battery those tests which do not agree with the criterion. The remaining set will then satisfy it. The reason for these difficulties is that Spearman's tetrad difference criterion demands more than his own hypothesis requires. His hypothesis does not state that there shall be only one common factor or ability. He himself deals with many factors. But his tetrad difference requires that there shall be only one common factor.

Now it happens that one can readily put together several batteries of tests such that *within each battery* the criterion is satisfied and therefore we have a common factor "*g*" in each battery. But if we take a few tests from each of these batteries and put together a new composite, then the criterion is not satisfied and we then require more than one factor. We are then laced with the ambiguity that we have several batteries of tests each with its own single common factor. Which of these common factors shall we call the general one? Which is it that we should call "*g*"? Spearman's answer is that we should use a set of perceptual tests as a reference and that if we are dealing with that particular common factor, then we should call it general but that if we are dealing with one of the other common factors, then we should call it a special ability, a group factor, a sub-factor of some sort. It would be more logical to assign a letter or a name to each of these mental abilities and to treat them as related dependent abilities. If we choose one of them, such as facility in dealing with perceptual relations, as an axis of reference (Spearman's "*g*")· then it should be frankly acknowledged that the choice is statistically arbitrary, for we could equally well start with verbal ability or with arithmetical ability as an axis of reference. The choice of the perceptual axis of reference might be made from purely psychological considerations, but it would not rest on statistical evidence.

The multi-dimensionality of mind must be recognized before we can make progress toward the isolation and description of separate abilities. It remains a fact, however, that since all mental tests are positively correlated, it is possible to describe the intercorrelations in terms of several factors in such a manner that one of the factors will be conspicuous in comparison with the others. But the exact definition of this factor varies from one set of tests to another. If it is this factor that Spearman implies in his theory of intelligence, then his criterion is entirely inadequate to define it, because the tetrad criterion merely tells us whether or not any given set of intercorrelations can be described in terms of one and only one common factor.

Let us start with the assumption that there may be several independent or dependent mental abilities, and let it be a question of fact for each study how many factors are needed to account for the observed intercorrelations.[1] We also make the assumption that the contributions of several independent factors are summative in the individual's performance on each one of the psychological tests. If we do not make this assumption, the solution seems well-nigh hopeless, and it is an assumption that is either explicit or implicit in all attempts to deal with this problem.

Reference: Thurstone, L. L. (1934). The vectors of the mind. *Psychological Review*, *41*, 1-32.

We may start our analysis of the generalized factor problem by considering the many hundreds of adjectives that are in current use for describing personalities and temperaments. We have made such a list. Even after removing the synonyms we still had several hundred adjectives. It is obvious at the start that all these traits are not independent. For example, people who are said to be congenial are also quite likely to be called friendly, or courteous, or generous, even though we do not admit that these words are exactly synonymous. It looks as though we were dealing with a large number of dependent traits.

The traditional methods of dealing with these psychological complexities have been speculative, bibliographical, or merely literary in character. The problem has been to find a few categories, called personality types or temperaments, in terms of which a longer list of traits might be described. Psychological inquiry has not yet succeeded in arriving at a list of fundamental categories for the description of personality. We are still arguing whether extraversion and introversion are scientific entities or simply artifacts, and whether it is legitimate even to look for any personality types at all.

In the generalized factor method we have one of the possible ways in which a set of categories for the scientific study of personality and temperament may be established on experimental grounds as distinguished from literary verbosity about this subject. It is our belief that the problem can be approached in several rational and quantitative ways and that they must agree eventually before we have a satisfactory foundation for the scientific description of personality.

The problem has geometrical analogies that we shall make use of. If we have a set of n points, defined by r coordinates for each point, we may discover that they are dependent in that some of these points can be described linearly in terms of the coordinates of the rest of the points. If the adjectives were represented by these points, it is as though we were to describe most of them in terms of a limited number of adjectives. But such a solution is not unique, because if we have ten points in space of three dimensions, then it is possible, in general, to describe any seven of the points in terms of the remaining three. It is just so with the personality traits, in that a unique solution is not given without additional criteria.

But before demanding this sort of reduction it would be of great psychological interest to know *how many* temperaments or personality types we must postulate in order to account for the differentiable traits useful. If we have a table of three coordinates for each one of ten points and if that matrix has the rank 2, then we know that all ten points lie in a plane and consequently they can all be described by two coordinates in that plane instead of by three. The application of this analogy would be the description of ten traits in terms of two independent traits which would then become psychological categories or fundamental types. They would constitute the frame of reference in terms of which the other traits would be described and in terms of which interrelations could be stated.

But since we have no given frame of reference to begin with, we do not have the coordinates of the points that might represent the adjectives. We therefore let each adjective represent its own coordinate axis, so that with *n* traits we shall have as many axes. These coordinate axes will be oblique, since the traits are known to be at least not all independent. The projection of any one of these traits *A* on the oblique axis through another trait *B* is the cosine of their central angle, but this is also the correlation between the two traits *A* and *B.* The correlations can be ascertained by experiment, and then our problem becomes that of finding the smallest number of orthogonal coordinate axes in terms of which we can describe all of the traits whose intercorrelations are known.

The actual data that we must handle are subject to chance errors. It is therefore profitable to see how the geometrical manner of thinking about this problem is affected by the chance errors. It can be shown that each test may be thought of as a vector in space of as many dimensions as there are independent mental factors. If the test is perfect, then it is represented geometrically by a unit vector. If the test has a reliability less than unity, then the length of the vector is reduced. In fact, the length of a test vector in the common factor space is the square root of its reliability. If the test has zero reliability, then it determines nothing and this fact has its geometrical correspondence in that the test vector is then a zero length so that it determines no direction at all in the space of mental abilities.

The obtained correlation between two tests is the scalar product of the two test vectors. If the two tests are perfect, then their scalars are both unity so that the true correlation between the two tests is the cosine of the angular separation between the two vectors.

	1	2	3
A	a_1	a_2	a_3
B	b_1	b_2	b_3
C	c_1	c_2	c_3
D	d_1	d_2	d_3
E	e_1	e_2	e_3
F	f_1	f_2	f_3
G	g_1	g_2	g_3

A	B	C	D	E	F	G
a_1	b_1	c_1	d_1	e_1	f_1	g_1
a_2	b_2	c_2	d_2	e_2	f_2	g_2
a_3	b_3	c_3	d_3	e_3	f_3	g_3

	A	B	C	D	E	F	G
A		r_{ab}	r_{ac}	r_{ad}	r_{ae}	r_{af}	r_{ag}
B	r_{ab}		r_{bc}	r_{bd}	r_{be}	r_{bf}	r_{bg}
C	r_{ac}	r_{bc}		r_{cd}	r_{ce}	r_{cf}	r_{cg}
D	r_{ad}	r_{bd}	r_{cd}		r_{de}	r_{df}	r_{dg}
E	r_{ae}	r_{be}	r_{ce}	r_{de}		r_{ef}	r_{eg}
F	r_{af}	r_{bf}	r_{cf}	r_{df}	r_{ef}		r_{fg}
G	r_{ag}	r_{bg}	r_{cg}	r_{dg}	r_{eg}	r_{fg}	

Figure 1. Factorial matrix A. Transpose of factorial matrix A'. Correlational matrix R. Factor theorem: $AA' = R$. Example: $r_{ab} = a_1b_1 + a_2b_2 + \cdots + a_rb_r$.

We shall consider next two of the fundamental theorems in a generalized theory of factors.[2] Let us take a table of seven variables as an example. In Figure 1 we have shown such a table toward the right side of the diagram. We may call it the correlational matrix. In a table of this kind we show the correlation of each test with every other test. For example, the correlation between the two tests *A* and *B* is indicated in the customary cell.

In the diagonal cells of a table of intercorrelations we are accustomed to record the reliabilities, but that is incorrect in factor theory unless the tests have been so chosen that they contain no specific factors. It is necessary for us to make a distinction between that part of the variance of a test which is attributable to the common factors and that part of the variance which is unique for each test. The part which is unique for each test may again be thought of as due to two different sources, namely, the chance errors in the test and the ability which is specific for the test. The reliability of a test is that part of the total variance which is due to the common factors as well as to the specific factor. It differs from unity only by that part of the variance which is due to chance errors. We need in factor theory another term to indicate that part of the total variance which is attributable only to the common factors and which eliminates not only the variance of chance errors but also the specific variance. We have used the term *communality* to indicate that part of the total variance of each test which is attributable to the common factors. It is always less than the reliability unless a specific factor is absent, in which case the communality becomes identical with the reliability. It is these communalities that should be recorded in the diagonal cells, but they are the unknowns to be discovered by the factorial analysis.

At the extreme left of the diagram we have a table of factor loadings. Here the seven tests are listed vertically and there are as many columns as there are factors. In the present example we have assumed three factors, so that we have three columns and seven rows. Let us suppose that the first factor represents verbal ability and that the second factor represents arithmetical ability. These particular abilities are not independent, but we may ignore that for the moment. Then the entry a_1 indicates the extent to which the test *A* calls for verbal ability and the entry a_2 indicates the extent to which it calls for arithmetical ability.

Since we have assumed three factors in this diagram we have three factor loadings for each test. A table in which the factor loadings are shown for each test we have called a *factorial matrix*, while the square table containing the intercorrelations we have called a *correlational matrix*.

The experimental observations give us the correlational matrix, so that it may be regarded as known. The object of a factorial analysis is to find a factorial matrix which corresponds to the given intercorrelations. There is a rather simple theoretical relation between the given correlational matrix and the factorial matrix. This relation constitutes the fundamental theorem of the present factor theory. It is illustrated by an example under the correlation table and it can also be stated in very condensed form by matrix notation in that *the factorial matrix multiplied by its transpose reproduces the correlational matrix* within the *observational* errors of the given correlation coefficients.

When we want to make an analysis of a table of intercorrelations, the first thing we want to know is how many independent common factors we must postulate in order to account for the given correlation coefficients. One of our fundamental theorems states that *the smallest number of independent common factors that will account exactly for the given correlation coefficients is the rank of the correlational matrix.* Although this theorem is of fundamental significance it is not possible to apply it in its theoretical form, because of the fact that the given coefficients are subject to experimental errors and the rank is therefore in general equal to the number of tests. It is always possible to account for a table of intercorrelations by postulating as many abilities as there are tests, but that is simply a matter of arithmetical drudgery and nothing is thereby accomplished. The only situation which is of scientific and psychological interest is that in which a table of intercorrelations can be accounted for by a relatively small number of factors compared with the number of tests.

There is no conflict between the present multiple factor methods and the tetrad difference method. When a single factor is sufficient to account for the given coefficients, then the rank of the correlational matrix must be 1, but the necessary and sufficient condition for this is that all of the second order minors shall vanish. Now if you expand the second order minors in a table of correlation coefficients you find that you are in fact writing the tetrads. Hence the tetrad difference method is a special case of the present multiple factor theorem.

It would be possible to extend the tetrad difference method by writing the expansions of the minors of higher order and in that manner to write formulae for any number of factors which correspond to the tetrads for the special case when one factor is sufficient. Such a procedure is unnecessarily clumsy. In fact, there should be no excuse for ever computing any more tetrads. Several better methods are available which give much more information with only a fraction of the labor that is required in the computation of tetrads.

Reference: Thurstone, L. L. (1934). The vectors of the mind. *Psychological Review*, *41*, 1-32.

TABLE 1 *List of Adjectives Used for Factor Study*

persevering	religious	haughty	eccentric
crafty	impetuous	submissive	ingenious
awkward	fickle	suspicious	accommodating
self-important	domineering	courageous	tactful
determined	frank	stern	careless
friendly	pessimistic	headstrong	tidy
patient	spiteful	jealous	precise
sarcastic	quiet	generous	systematic
congenial	disagreeable	dependable	cheerful
hard-working	reserved	faithful	conscientious
stubborn	refined	reserved	grasping
capable	unnatural	solemn	satisfied
tolerant	bashful	earnest	cynical
calm	self-reliant	talented	courteous
peevish	broad-minded	frivolous	unconventional
			quick-tempered

We return now to the multiple factor analysis of personality. In Table 1 we have a list of sixty adjectives that are in common use for describing people. These adjectives together with their synonyms were given to each of 1300 raters. Each rater was asked to think of a person whom he knew well and to underline every adjective that he might use in a conversational description of that person. Since it was not necessary for the rater to reveal the name of the person he was rating it is our belief that the ratings were relatively free from the inhibitions that are usually characteristic of such a task. With 1300 such schedules we determined the tetrachoric correlation coefficient for every possible pair of traits. Since there were sixty adjectives in the list we had to determine 1770 tetrachoric coefficients. For this purpose we developed a set of computing diagrams which enable one to ascertain the tetrachoric coefficients with correct sign by inspection.[3] Each coefficient can be ascertained in a couple of minutes by these computing diagrams.

The table of coefficients for the sixty personality traits was then analyzed by means of multiple factor methods[4] and we found that five factors are sufficient to account for the coefficients. We reproduce in Figure 2 the distribution of discrepancies between the original tetrachoric coefficients and the corresponding coefficients that were calculated by means of the five factors. It has a standard deviation of .069. The average standard error of thirty tetrachoric coefficients chosen at random in this table is .052.

Reference: Thurstone, L. L. (1934). The vectors of the mind. *Psychological Review*, *41*, 1-32.

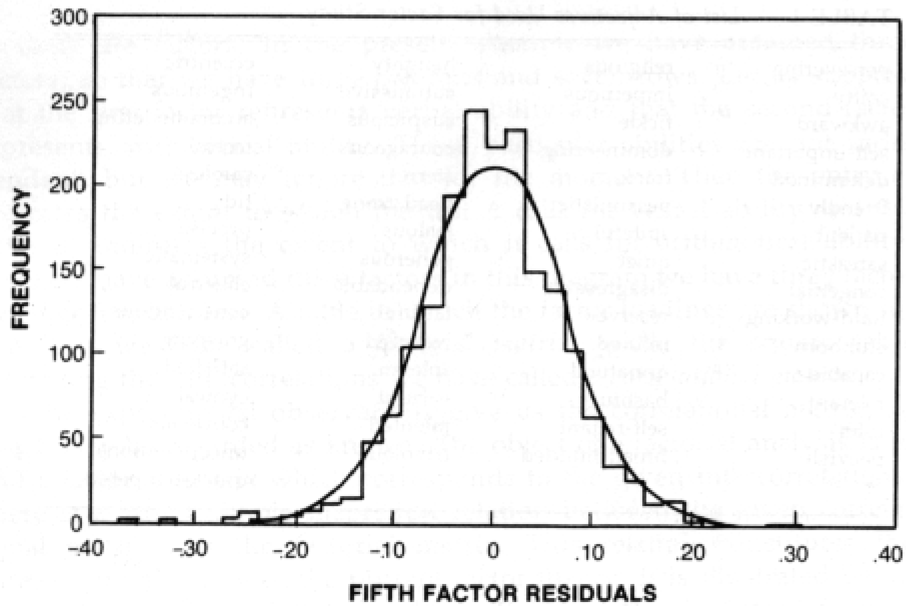

Figure 2. Frequency distribution of fifth factor residuals in a study of sixty personality traits.

It is of considerable psychological interest to know that the whole list of sixty adjectives can be accounted for by postulating only five independent common factors. It was of course to be expected that all of the sixty adjectives would not be independent, but we did not foresee that the list could be accounted for by as few as five factors. This fact leads us to surmise that the scientific description of personality may not be quite so hopelessly complex as it is sometimes thought to be.

Next comes the natural question as to just what these five factors are, in terms of which the intercorrelations of sixty personality traits may be described. Each of the adjectives can be thought of as a point in space of five dimensions and- the five coordinates of each point represent the five factor components of each adjective.

We shall consider a three-dimensional example in order to illustrate the nature of the indeterminacy that is here involved (Figure 3). Let us suppose that three factors are sufficient to account for a list of traits. Then each trait can be thought of as a point in space of three dimensions. In fact, each trait can be represented as a point on the surface of a ball. If two traits *A* and *B* tend to coexist, the two points will be close together on the surface of the ball. If they are mutually exclusive, so that when one is present the other is always absent and vice versa, then the two traits are represented by two points that are diametrically opposite on the surface of the ball such as *A* and D. If the two traits are independent and uncorrelated, such as *A* and C, then they will be displaced from each other in the same way as the north pole and a point on the equator, namely by ninety degrees.

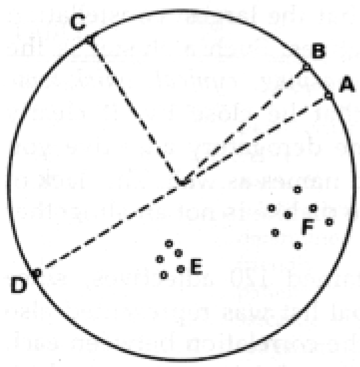

Figure 3. Correlation *AB* is high and positive. Correlation *AC* is zero. Correlation *AD* is −1. *E* and *F* represent constellations. Location of the orthogonal axes is arbitrary.

Now suppose that the traits have been allocated to points on the surface of the sphere in such a manner that the correlation for each pair of traits agrees closely with the cosine of the central angle between the corresponding points on the surface of the sphere. Then we want to describe each of these traits in terms of its coordinates, but we should first have to decide where to locate at least two of the three coordinate axes. This is an arbitrary matter, because the internal relations between the points, that is, the intercorrelations of the traits, remain exactly the same no matter where in the sphere we locate the coordinate axes. That is, when the points have been assigned on the surface of the ball, we have not thereby located the respective coordinate axes. It is possible to determine uniquely how many independent common factors are required to account for the intercorrelations without thereby determining just what the factors are. We may therefore use any arbitrarily chosen set of orthogonal axes.

It is psychologically more illuminating to investigate constellations of the traits. In the factor analysis of the adjectives, constellations of traits reveal themselves in that the points which represent some of the traits lie close together in a cluster on the surface of a five-dimensional sphere. One or two examples will illustrate this type of analysis with respect to the personality traits. We find, for example, that the following traits lie close together in a duster, namely, *friendly, congenial, broad-minded, generous,* and *cheerful.* It would seem, therefore, that as far as the five basic factors are concerned, whatever be their nature, these several traits are very much alike as far as can be determined by the way in which we actually use these adjectives in describing people.

Another such cluster of adjectives which are used as though they signified the same fundamental trait are the adjectives *patient, calm, faithful,* and *earnest.* They cling close together in the factorial analysis. Another small group is found in the three traits *persevering, hard-working,* and *systematic,* which lie close together. Still another one is the cluster of traits *capable, frank, self-reliant,* and *courageous.*

It is of psychological interest to note that the largest constellation of traits consists of a list of derogatory adjectives. Such a cluster is the following: *self-important, sarcastic, haughty, grasping, cynical, quick-tempered,* and several other derogatory traits that lie close by. It clearly indicates that if you describe a man by some derogatory adjective you are quite likely to call him by many other bad names as well. This lack of objectivity in the description of the people we dislike is not an altogether unknown characteristic of human nature.

The schedules used in this study contained 120 adjectives, since every one of the 60 adjectives in the principal list was represented also by a synonym. This enabled us to ascertain the correlation between each pair of synonyms and we used it as an index of the consistency with which the trait was judged. Let us consider this correlation to be an estimate of the reliability of the judgments of the trait. By factor analysis we know the communality of each trait. It is that part of its total variance which it has in common with the other adjectives, namely, the five common factors. The difference between these two variances is the specificity. It shows the magnitude of the specific factor in each adjective.

We have listed these differences for each adjective in Table 2 and they yield some psychologically interesting facts. We find, for example, one adjective which has a surprisingly high specificity of nearly .60, and we want to know, of course, what kind of trait it represents. We find that it is the adjective *talented.* It seems reasonable to guess that this adjective refers largely to the intellectual abilities which are not represented by this list. When this study is repeated, we shall include several adjectives of this type, so that intelligence may be investigated as a vector in relation to the personality traits.

Another item with a high specificity is the adjective *awkward.* This means that the trait *awkward* has something about it which is unique in our list of sixty and which is not represented by the five common factors. This trait is probably ease and facility in body movement, which is certainly not represented in the rest of the sixty traits. When this study is repeated we may or we may not include several adjectives of this type, depending on whether we want to include this additional factor in a study of personality. Another adjective with high specificity is *religious* and the explanation is undoubtedly along the same lines because this is the only adjective in the list that refers to any kind of religiousness.

Studies of this sort should be repeated until every important trait is represented by several adjectives. The analysis should yield as many independent factors as may be required.

When the factorial analysis is complete, the specifics should all vanish or they should be relatively small. Then the communalities and the reliabilities will have nearly the same value. The constellations to be found in such an analysis will constitute the fundamental categories in terms of which a scientific description of personality may be attained.

There is no necessary relation between the number of factors and the number of constellations. A system of tests might be found which can be accounted for by several factors even though it contains no constellations. Fortunately the constellations can be isolated in a very simple manner when the coefficients have been corrected for attenuation.

TABLE 2

Specificity	Adjective	Specificity	Adjective
.18	persevering	.05	broad-minded
.40	crafty	.00	haughty
.46	awkward	.07	submissive
.10	self-important	.17	suspicious
.16	determined	.27	courageous
.14	friendly	.01	stern
.02	patient	.01	headstrong
.16	sarcastic	.25	jealous
.16	congenial	.16	generous
.34	hard-working	.17	dependable
.34	stubborn	.13	faithful
.16	capable	.00	solemn
.04	tolerant	.04	earnest
.27	calm	.59	talented
.09	peevish	.39	eccentric
.58	religious	.23	ingenious
.36	impetuous	.30	accommodating
.07	fickle	.26	tactful
.19	domineering	.34	tidy
.40	frank	.22	precise
.22	pessimistic	.22	systematic
.04	spiteful	.03	cheerful
.12	quiet	.11	conscientious
.19	disagreeable	.30	grasping
.08	reserved	.68	satisfied
.27	refined	.29	cynical
.18	unnatural	.12	courteous
.48	bashful	.14	unconventional
.14	self-reliant	.41	quick-tempered

I turn next to a factor study of the insanities. I have used a very elaborate set of data which Dr. Thomas Verner Moore of Washington, D.C., collected and investigated by other factor methods. Dr. Moore worked with a list of forty-eight symptoms, thirty-seven of which are listed in Table 3. He recorded the presence-absence, or a rating or test measure of each symptom for each of several hundred patients. With these records it was possible to ascertain to what extent any two symptoms tend to coexist in the same patient. For example, the extent to which the two symptoms *excited* and *destructive* tend to coexist in the same patient is indicated by the tetrachoric correlation of +.71. The records were sufficiently complete so that we could prepare a table of intercorrelations of the tetrachoric form for thirty-seven symptoms or 666 coefficients. These computations were also made by the computing diagrams.

TABLE 3 *Multiple Factor Analysis of Psychotic Symptoms*

Code No.	Psychotic Symptom	Code No.	Psychotic Symptom
1.	Alcoholism of Parents	28.	Memory Ratio
2. •	Anxious	29.	Mutism
3.	Attacks, Number of Previous	30.	Negativism
6.	Delusions, Bizarre	31.	Neurasthenia
9.	Depression	32.	Perception Defect
10.	Destructive	33.	Reasoning
11.	Disorientation in Space	34.	Refusal of Food
13.	Euphoria	35.	Retardation
14.	Excited	36.	Sensibilities, Loss of Finer
15.	Fallacies, Autistic	37.	Shut In
16.	Fallacies, Logical	38.	Stereotypism of Actions
17.	Giggling	39.	Stereotypism of Any Kind
18.	Hallucinations, Auditory	40.	Stereotypism of Attitudes
19.	Hallucinations, Others	41.	Stereotypism of Words
23.	Homicidal	42.	Suicidal
24.	Insane Relatives	43.	Tantrums
25.	Insight, Absence of	44.	Tearful
26.	Irritable	45.	Voices, Speaking to
27.	Memory, Total Defect		

References: (1) Moore, T. V., *Amer. J. Psychiat.*, 1930, 9, 719-738. (2) Moore, T. V., Series of Research Publications of *Ass'n for Research in Nervous and Mental Diseases*, Vol. 9, 324-339; Baltimore, Williams and Wilkins.

The multiple factor method was then applied to the table of 666 coefficients and we found that five factors are sufficient to account for the correlations, with residuals small enough so that they can be ignored. The communalities were then computed for each one of the symptoms and we found that about ten of the symptoms do not contain enough in common with the other symptoms to warrant their retention in a factor study. In other words, about ten of the symptoms are either so specific in character or so unreliable as to estimates that they do not yield significant correlations with the several other symptoms. This left twenty-six symptoms which are more or less related and for which the factorial clusters of symptoms could be profitably investigated.

In Table 4 we have listed the psychotic symptoms which lie in each of several constellations. We find, for example, that the following symptoms are functionally closely related, namely, *mutism, negativism, being shut-in, stereotypism of action, stereotypism of attitudes. Stereotypism of words,* and *giggling.* These seven traits are evidently related in that they tend to be found in the same patients and we recognize the list as descriptive of the catatonic group. Another constellation consists in the presence of *logical fallacies, defect in memory, defect in perception,* and *defect* in *reasoning.* This is a constellation of symptoms that indicates a derangement of the cognitive functions of the patient as contrasted with derangements of the affective aspects of his mentality. Another group of three symptoms that lie close together in the factorial analysis consists of the traits *destructive, irritable,* and *having tantrums.* A fourth cluster contains the symptoms *delusions,* auditory and other types of *hallucinations,* and *speaking* to *voices.* A fifth group contains the symptoms *anxious, depressed,* and *tearful.*

TABLE 4 *Psychotic Symptoms*

Cluster A, Catatonic Group	*Cluster B, Cognitive Group*
Mutism	Logical fallacies
Negativism	Total memory defect
Shut in	Perception defect
Stereotypism of action	Reasoning
Stereotypism of attitudes	Disorientation in space
Stereotypism of words	
Giggling	
Cluster C, Manic Group	*Cluster D, Hallucinatory Group*
Destructive	Bizarre delusions
Excited	Auditory hallucinations
Irritable	Other hallucinations
Tantrums	Speaking to voices
Cluster E, Depression	
Anxious	
Depression	
Tearful	
Retardation in movement	

It is not likely that this analysis of the tetrachoric correlations between symptoms has given us anything like a dependable classification of the insanities, because the study can be much improved when it is done the next time. But our results indicate that by the multiple factor methods it should be possible to arrive at a rational classification of the insanities and of personality types.

Another application of the present factor methods is an analysis to ascertain whether the vocational interests of college students can be classified in constellations and whether they can be described in terms of vocational interest types, small in number compared with the list of available occupations. These should eventually be related to the temperamental and personality traits and to the constellations of mental abilities.

We asked three thousand students in four universities to indicate their likes and dislikes on a list of eighty of the better known occupations (Table 5) which are available for college students. The tetrachoric correlations indicate the extent to which those who are interested in engineering, for example, can also be expected to have some interest in physics, or in chemistry. Those who are interested in engineering tend to dislike law and journalism and salesmanship, so that these correlations are negative. Our first question will be to ascertain the number of factors that are necessary to account for the observed intercorrelations. Previous inquiry indicates that the number of types may not exceed six or eight.

TABLE 5

Actor	Explorer	Pharmacist
Advertiser	Factory Manager	Philosopher
Architect	Farmer	Photographer
Army Officer	Florist	Physician
Art Critic	Foreign Correspondent	Physicist
Artist	Forest Ranger	Poet
Astronomer	Fruit Grower	Press Agent
Athletic Director	Geologist	Printer
Auctioneer	High School Teacher	Private Secretary
Auto Salesman	Historian	Professional Athlete
Banker	Insurance Salesman	Psychologist
Biologist	Inventor	Public Speaker
Botanist	Jeweler	Radio Announcer
Building Contractor	Journalist	Railway Conductor
Business Manager	Judge	Real Estate Salesman
Cattle Rancher	Landscape Gardener	Retail Merchant
Certified Public Accountant	Lawyer, Criminal	Sales Manager
Chemist	Lawyer, Corporation	Scientist
Civil Engineer	Librarian	Sculptor
Clergyman	Manufacturer	Secret Service Man
Club Secretary	Mathematician	Ship Officer
College Professor	Mechanical Engineer	Sociologist
Congressman	Musician	Stockbroker
Dentist	Newspaper Reporter	Surgeon
Diplomatic Service	Novelist	Tax Expert
Economist	Office Manager	Vocational Counselor
Electrical Engineer	Orchestra Conductor	

The scoring of the individual schedules might be reduced to a number of scores equal to the number of factors. These scores would be the coordinates of a point which represent the subject in a space of as many dimensions as there are factors required by the intercorrelations. The occupational likes and dislikes of the subject might be estimated by the coordinates of the point which represent his own interests. In this manner it may be possible to make a limited list of occupations that are typical of the interests of each student.

The question might be raised whether a student should be advised to enter that occupation for which his interests are typical. It might well be argued that he has a better chance of success if he enters a profession for which his interests are unusual. But that question refers, of course, to educational and vocational guidance while our present problem concerns the methodology of isolating types or constellations of traits. If we should find that vocational interests group themselves into a relatively small number of types it would have psychological interest in relation to the personality traits and mental abilities of the same subjects. It would be another question to decide how such data could or should be used in guidance.

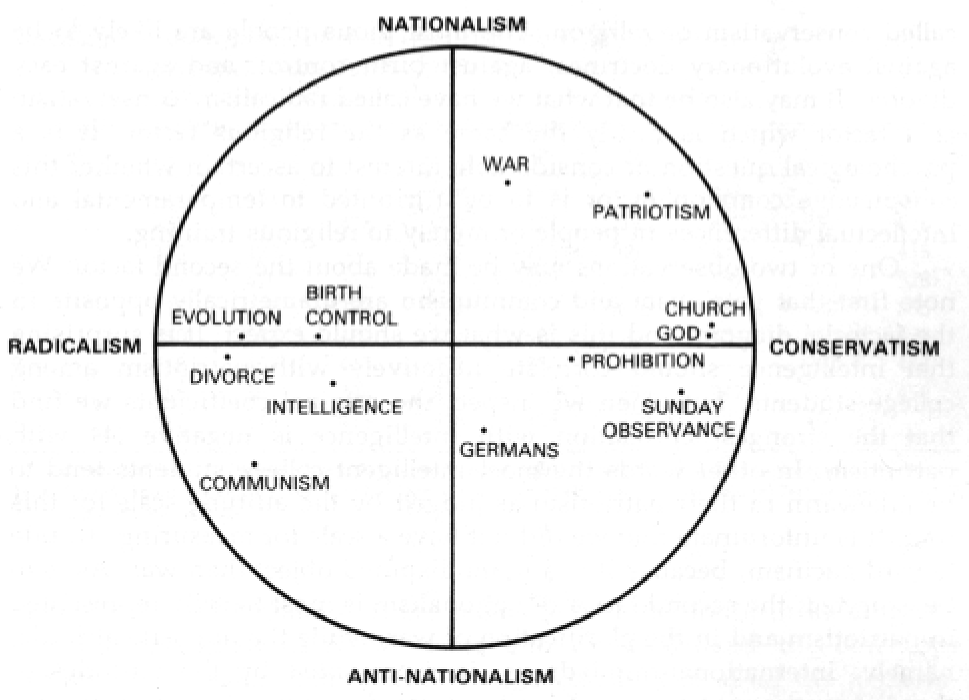

Figure 4. Factor study of radicalism with attitude scales by Thelma Gwinn Thurstone.

An application of the factor methods has recently been made by Mrs. Thurstone. Her problem was to ascertain whether radicalism is a common factor in people's attitudes on various disputed social issues. She gave eleven attitude scales to about 380 students in several universities. Records of the intelligence examination of the American Council on Education were available for the majority of these students, so that the study includes twelve variables. The factorial analysis reveals a conspicuous common factor that we recognize as radicalism. The second factor residuals were comparable with the standard errors of the given coefficients. The standard error of the mean correlation coefficient was .047 while the standard error of the second factor residuals was .056.

Since the two factors were sufficient to account for the intercorrelations, we have plotted in Figure 4 the factor loadings as the two coordinates for each of the twelve variables. Several psychological interpretations can be made from this diagram. The variables which are heavily loaded with radicalism are attitudes favorable to evolutionary doctrines, favorable to birth control, favorable to easy divorce, favorable to communism, and it is of interest to note that intelligence is positively correlated with these radical or liberal attitudes. In the opposite direction we find conservatism in attitudes favorable to the church, favorable to prohibition, the observance of Sunday, and belief in a personal Cod. Inspection of the original coefficients as well as the factorial analysis shows the more intelligent college students tend to be radical or liberal on social questions and that they tend to be atheistic or agnostic on religious matters.

In naming the common factor which is most prominent in these attitude scales there may be some question as to whether it should be called conservatism or religion. The most pious people are likely to be against evolutionary doctrines, against birth control, and against easy divorce. It may also be that what we have called radicalism- conservatism is a factor which is nearly the same as the religious factor. It is a psychological question of considerable interest to ascertain whether this conspicuous common factor is to be attributed to temperamental and intellectual differences in people or merely to religious training.

One or two observations may be made about the second factor. We note first that patriotism and communism are diametrically opposite in the factorial diagram and this is what we should expect. It is surprising that intelligence should correlate negatively with patriotism among college students; but when we inspect the original coefficients we find that the strongest correlation with intelligence is negative .44 with patriotism. In other words the most intelligent college students tend to be lukewarm in their patriotism as judged by the attitude scale for this trait. It is unfortunate that we did not have a scale for measuring attitude toward pacifism, because it is a more disputed object than war. As is to be expected, the second factor of nationalism is most heavily represented in patriotism and in the glorification of war, while the opposite attitude, namely, international-mindedness, is represented by the attitudes of those Americans who are friendly toward communism and toward Germany. These examples will suffice to illustrate the possibilities of factorial analysis in dealing with the affective and temperamental attributes of people.

Until recently practically all of the studies that have been made by factor methods have been confined to the cognitive traits and especially to the mental abilities that are represented by psychological tests. It is on this type of material that Professor Spearman and his students have worked extensively with the tetrad difference method. I shall describe the analysis of a set of nine psychological tests which have been investigated by Mr. W. P. Alexander of the University of Glasgow and I reproduce here with his permission a section of his unpublished data (Table 6). The nine tests are all well known except the new performance test which he has recently devised. When we apply the multiple factor methods to this correlational matrix we find that most of the variance of each test can be accounted for by postulating only two factors. These we have plotted in Figure 5.

TABLE 6 *Correlations of Nine Tests Used by W. P. Alexander*
Westfield State Farm—71 Cases
Correlation Matrix

Test	1	2	3	4	5	6	7	8	9
1	—	451	373	337	717	750	768	786	612
2	451	—	428	470	277	393	407	557	616
3	373	428	—	362	255	402	415	334	415
4	337	470	362	—	189	304	257	203	528
5	717	277	255	189	—	695	752	800	335
6	750	393	402	304	695	—	834	850	478
7	768	407	415	257	752	834	—	917	433
8	786	557	334	203	800	850	917	—	450
9	612	616	415	528	335	478	433	450	—

Description of Tests: (1) Stanford-Binet; (2) Pintner-Paterson Scale; (3) Healy Puzzle II–Picture Completion; (4) Porteus Maze Test; (5) Thorndike Reading Test; (6) Otis Group Test; (7) Otis Self-Administering Test; (8) Terman Group Test; (9) Alexander Performance Scale.

In this diagram the abscissae represent the first factor that was extracted and the ordinates represent the second factor. As is to be expected the centroid of the whole system is on the *x-axis* since this is implied in the approximation procedure that was used for this problem. Even in a first glance at this diagram we are struck by the fact that the nine tests divide themselves into two groups or clusters. We turn immediately to the tests to see what psychological abilities may be found in the tests that constitute each of these two constellations. We then find that all of the verbal tests fall in one of these constellations and that all of the performance tests fall in the other constellation. This suggests a rational method of establishing the psychological categories that we should call *mental abilities.* There seems to be little doubt in naming one of these constellations *verbal ability* and we shall name the other one tentatively as *manipulation,* since that is common to all of the performance tests. The verbal tests that cling together in this analysis are the Stanford-Binet, the Thorndike reading test, the Otis group test, the Otis self-administering test, and the Terman group test. The performance tests that group themselves apart from the verbal tests are the Pintner-Paterson scale, the Healy Puzzle II (picture completion), the Porteus maze test, and Alexander's new performance test.

Reference: Thurstone, L. L. (1934). The vectors of the mind. *Psychological Review*, *41*, 1-32.

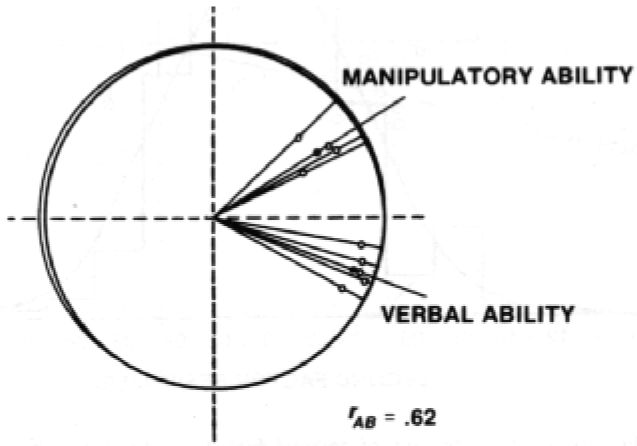

Figure 5. Two factor analysis of nine tests used by W. P. Alexander.

It is to be noted that these two constellations are entirely independent of the location of the two orthogonal axes through this system of nine tests. A characteristic of the multiple factor problem is that the location of the axes is arbitrary and that hence the factorial components are to that extent arbitrary and without fundamental psychological significance. This limitation is entirely obviated if we center our interest on the constellations of mental traits that are revealed in the factorial analysis. The relations between the constellations are invariant under rotation of the orthogonal axes, and hence we have here something more or less permanent in terms of which we may define psychological categories and mental abilities.

As illustrative of this method we have found the centroid of each of the two constellations in Alexander's nine tests. These centroids are indicated by the small black circles. If we had a fairly large number of tests in each constellation we could attach some confidence to the centroid of the constellation as a definition of a mental ability. I am here using the term *mental ability* to identify a constellation of tests which lie in the same cluster in the factorial analysis and which correlate nearly unity when they are corrected for attenuation.

In Alexander's data we find that the two constellations are not independent. This is seen on the diagram by the fact that the central angle between the two constellations is not a right angle. Since all mental abilities are positively correlated, we should expect that all of the constellations of mental abilities will be positively correlated also, and that is the case in the present data. Now if we are to use these constellations of traits as the categories for psychological description and if we are to define mental abilities and personality types in terms of constellations, then it is immediately apparent that our categories will not all be independent. This indicates that we must look eventually to still more fundamental categories in terms of which to describe the mental abilities. In the meantime it will be useful to describe them in terms of constellations of known degrees of dependence and we shall then be using a system of oblique coordinates instead of the orthogonal coordinates of conventional mathematics. Ultimately we shall want to find that particular location of the orthogonal axes which corresponds to independent genetic elements.

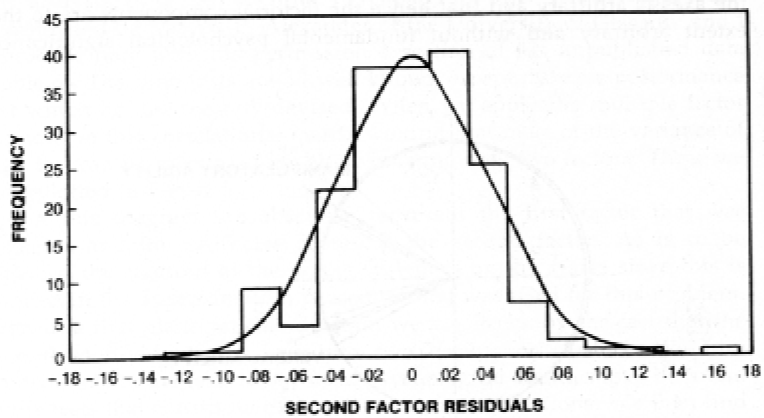

Figure 6. Frequency distribution of second factor residuals for 20 tests used by Dr. William Brown. "A test of the theory of two factors," *Brit. J. Psychol.*, April, 1933, Approximate solution by multiple factor methods.

I shall now describe briefly the results of a multiple factor analysis of a set of twenty tests that were recently used by Dr. William Brown in support of the Spearman single-factor hypothesis of intelligence. We can sympathize with Dr. Stephenson who computed all of the fifteen thousand tetrad differences for this table and we shall not even venture to guess how long it must have taken to make the computations.

The multiple factor method in its simplest approximation form was applied to this set of data, and we find that two factors are sufficient to account for the given coefficients. The distribution of second factor residuals is shown in Figure 6. The standard deviation of the distribution of residuals is .039. In Figure 7 we have plotted the two-factor coordinates for these twenty tests and we then see that they fall into two constellations. Reference to the tests shows that all of the verbal tests are in one group and that all of the perceptual tests are found in the other group. We might regard these two groups as representing two mental abilities, one of which we should call verbal ability, while the other one might be called visual farm perception. The correlation between pure measures of these two abilities is the cosine of the angular separation between the two centroids.

Reference: Thurstone, L. L. (1934). The vectors of the mind. *Psychological Review*, *41*, 1-32.

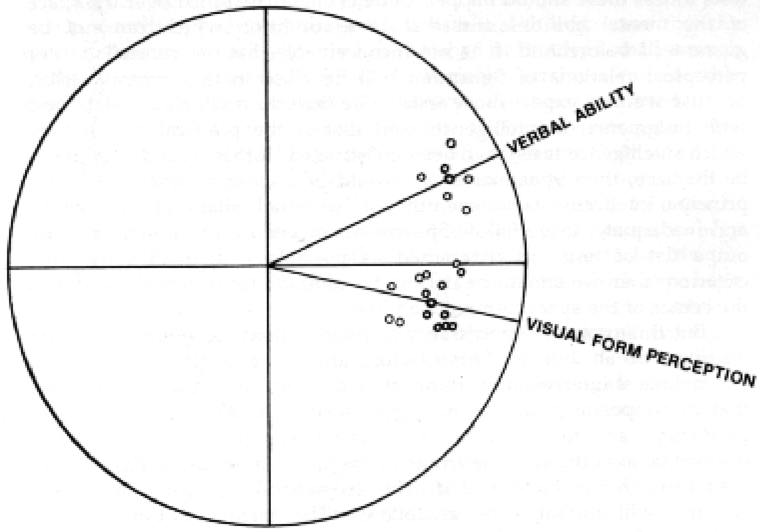

Figure 7. Two-factor analysis of the 20 tests used by Dr. William Brown in "A test of the theory of two factors," *Brit. J. Psychol.*, April, 1933.

Brown's interpretation of these relations which he has investigated with the tetrad difference method is that one of these two clusters represents Spearman's "*g*" and that whatever is left over is attributable to verbal ability. It seems to us that it would be just as logical to call the verbal cluster the general one and to attribute the residuals to a special perceptual factor. In that case the verbal constellation would be called the general "*g*." This reasoning might be extended to each one of the many constellations of abilities which are represented by groups of psychological tests. But the evidence is as yet entirely insufficient to demonstrate that any one of the constellations of mental abilities is a principal intellective factor.

Since all mental tests are positively correlated, it follows that the tests must lie in a limited quadrant or octant of the geometrical representation of the mental abilities. Instead of adopting one of the constellations as a reference it might be more logical to find a point in the octant of the mental abilities such that the most diversified series of tests all correlate positively with the imaginary test at this central point. It is not inconceivable that a point might be located by this means with some degree of certainty. It would then represent the central intellective factor and those tests would be the best tests of intelligence which correlate highest with the imaginary pure test, which is represented by this central vector. It would not be the centroid of any particular set of tests unless these should happen to be evenly distributed over the space of the mental abilities and that is a condition which can not be guaranteed beforehand. It is not inconceivable that the constellation of perceptual relations of Spearman will lie close to this central vector, because we may expect those tests to lie close to it which correlate best with judgments of intelligence, and that is the practical criterion by which intelligence tests have been constructed. If that should turn out to be the case, then Spearman's "*g*" would be a close representation of a principal intellective factor even though his tetrad criterion is ambiguous and inadequate. If we follow Spearman's procedure in carefully picking out a list of tests, as diversified as possible, which all satisfy his criterion, then we should be defining a common factor which is close to the center of the space of mental abilities.

But this procedure necessarily leads to at least some negative factor loadings in all but the first factor, and it is difficult to make a psychological interpretation of negative abilities. It is much more likely that the opposite procedure will give a solution that makes sense for psychology and for genetics. This solution would be to find a set of orthogonal axes through the fringe of the space of mental abilities rather than through the middle of it. The geometrical representation of the solution will probably be as follows. The mental abilities can be represented as points within a cone. The axis of the cone will represent a fictitious central intellective factor. The fundamental abilities which have genetic meaning will be represented by a set of orthogonal elements of the cone in space of as many dimensions as there are genetic factors. All mental tests will then be described in terms of positive orthogonal coordinates, corresponding to the independent genetic factors. Negative loadings will disappear. For rough and practical descriptive purposes the axis of the cone of mental abilities may be used as an axis of reference which is central in the space of mental abilities. It will be a fictitious central intellective factor but it will probably have no fundamental psychological or genetic meaning.

In conclusion I want to suggest the course of investigation which is likely to lead to a scientific description and understanding of mental abilities and personality traits and their aberrations. Our first task is to establish the identity of the several mental abilities which reveal themselves as distinct constellations in factorial analysis. Among these abilities it is quite likely that we shall find verbal ability, perceptual relations, and arithmetical ability to be distinct, though positively correlated.

In these studies it is probably best not to pivot on any single constellation as fundamental to all of the rest. It is better to use an analysis which allows as many factors to appear as are necessary to account for each new set of tests and to name the constellations when they appear. These categories should be frankly regarded as temporary and subject to redefinition in successive experiments. Eventually we should be able to work with a rather limited number of mental abilities and trait constellations. These categories will be either more or less conventionalized, or else they will be so consistent that some physical or even genetic significance may be attached to them.

In extracting each constellation it is essential to make sure that the specific variance in each variable has vanished or that it is small enough so that it can be ignored. If one of the variables retains a considerable specific variance after the common factors have been extracted, it is necessary to repeat the experiments with additional tests which are similar to the one which has shown a specific variance. Only when the reliability and the communality are nearly equal can we be sure that the common factors account for the total variance in each test. In order to favor this outcome it is best to assemble the test batteries in such a way that there are several similar tests of each kind that are to be investigated. This is in a sense the opposite of the precautions which have been current in factor studies where the experimenters have been careful to avoid similar tests simply because they disturb the tetrad criterion. Instead of concealing the specific variance with the chance errors, as is done in tetrad analysis, it is more illuminating to investigate the nature of the specific variance of each test by including several similar tests of each kind in the test battery.

When the mental abilities have been defined in terms of a large number of elementary tests for each ability, it will be of considerable interest to ascertain experimentally the extent to which training of one ability transfers to another ability and to relate such transfer effects to the known correlations between the abilities.

It is my conviction that the isolation of the mental abilities will turn out to be essentially a problem in genetics. It will be profitable to ascertain what part of the variance of each ability can be attributed to the parents. It is not unlikely that this type of genetic research will constitute one of the means for identifying the independent elements in the mental abilities.

We hope that the development of factorial methods of analysis will give us the tools by which to reduce the complexities of social and psychological phenomena to a limited number of elements. These methods should be useful not only for developing the theory of mental abilities and temperamental traits but also in meeting the practical demands of educational and psychological guidance.

Footnotes

1. This point of view is represented by the work of T. L. Kelley as distinguished from the work of Spearman and his students.
2. The present generalized factor theory has been described in two lithographed pamphlets by the writer. These are: "The theory of multiple factors," The University

of Chicago Bookstore, Chicago, Ill., and "A simplified multiple factor method," The University of Chicago Bookstore.

3. "Computing diagrams for the tetrachoric correlation coefficient," The University of Chicago Bookstore.
4. "A simplified multiple factor method," Univ. Chicago Bookstore, 1933.