Answers to Problems

1. a. To estimate the costs, calculate the number of square feet in the exterior of the house and enter that amount into the equation.

Calculate the number of square feet in the exterior of the house:
2(9x40)+ 2(9x28) = 1,224

Estimated Cost = $15 + ($0.06/s.f.)(1,224 s.f.)

= $88.44

b. If the cost per square increases, the new cost should be used to estimate costs.

Estimated Cost = $15 + ($0.064/s.f.)(1,224 s.f.)

= $93.34

c. The amount of time it takes to do the job could be affected by the type of siding on the house and whether the house is one or two stories. The type of siding might also affect the amount of cleaning supplies needed. Another factor that could affect costs is the distance that a crew must travel to get to a job.

3.

PERIOD	CODE (X)	BEEF IMPORTED(Y)	(X)(Y)	X^2
2003	1	82	82	1
2004	2	101	202	4
2005	3	114	342	9
2006	4	126	504	16
2007	5	137	685	25
2008	6	151	906	36
2009	7	164	1,148	49
2010	8	182	1,456	64
2011	9	189	1,701	81
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SUM	45	1,246	7,026	285

$$b = \frac{n\Sigma(X)(Y) - \Sigma X\ \Sigma Y}{n\Sigma X^2 - (\Sigma X)^2}$$

$$= \frac{(9)(7,026) - (45)(1,246)}{(9)(285) - (45)^2}$$

= 13.27 thousands of tons of beef/year

$$a = \frac{\Sigma Y}{n} - \frac{b(\Sigma X)}{n}$$

$$= \frac{1,246}{\quad} - \frac{13.27(45)}{\quad}$$

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= 72.1 thousands of tons of beef

Imports of beef are projected to increase 13.27 thousands of tons per year. The interception is 72.1 thousands, and it represents the point on the regression line for 1992.

$$Y_e = 72.1 + 13.27(X)$$

The periods are consecutively numbered, 2012 is coded as 10, and thus, the projected amount of beef imported for year 2012 is:

$$Y_{12} = 72.1 + (13.27)(10)$$
$$= 204.8 \text{ thousands of tons}$$

5. a. To calculate the MSE and the MAD, the forecasted amounts for December through July must be calculated. The parameters for this model are calculated in the previous problem.

The periods are consecutively numbered, December is coded as 1, so, the projected number of hours for December is:

$$Y_{De} = 291.1 + (158.1)(1)$$
$$= 449.2 \text{ hours}$$

$$Y_{Ja} = 291.1 + (158.1)(2)$$
$$= 607.3 \text{ hours}$$

$$Y_{Fe} = 291.1 + (158.1)(3)$$
$$= 765.4 \text{ hours}$$

$$Y_{Mr} = 291.1 + (158.1)(4)$$
$$= 923.5 \text{ hours}$$

$$Y_{Ap} = 291.1 + (158.1)(5)$$
$$= 1{,}081.6 \text{ hours}$$

$$Y_{My} = 291.1 + (158.1)(6)$$
$$= 1{,}239.7 \text{ hours}$$

$$Y_{Ju} = 291.1 + (158.1)(7)$$
$$= 1{,}397.8 \text{ hours}$$

$$Y_{Jy} = 291.1 + (158.1)(8)$$
$$= 1{,}555.9 \text{ hours}$$

PERIOD	**ACTUAL HRS OF SERVICE**	**FORECASTED HRS OF SERVICE**	**ERROR**	**SQUARED ERROR**	**ABSOLUTE ERROR**
December	300	449.2	-149.2	22,260.64	149.2
January	750	607.3	142.7	20,363.29	142.7
February	650	765.4	-115.4	13,317.16	115.4
March	920	923.5	- 3.5	12.25	3.5
April	1,300	1,081.6	218.4	47,698.56	218.4
May	1,400	1,239.7	160.3	25,696.09	160.3
June	1,200	1,397.8	-197.8	39,124.84	197.8
July	1,500	1,555.9	- 55.9	3,124.81	55.9
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SUM			- .4	171,597.64	1,043.2

$$MSE = \frac{\sum_{t=1}^{n} (x_t - f_t)^2}{n}$$

$$= \frac{171,597.64}{8}$$

$$= 21,449.7$$

$$MAD = \frac{\sum_{t=1}^{n} |x_t - f_t|}{n}$$

$$= \frac{1,043.2}{8}$$

$$= 130.4$$

b. The model provides reasonably good predictions based on the MSE and MAD. The MAD is easier to interpret because it is an estimate of the average error in the forecast.

7. a. When A = .1

$f_t = A\,(x_{t-1}) + (1-A)\,f_{t-1}$

For the initial value, we assume $f_{01} = x_{00}$

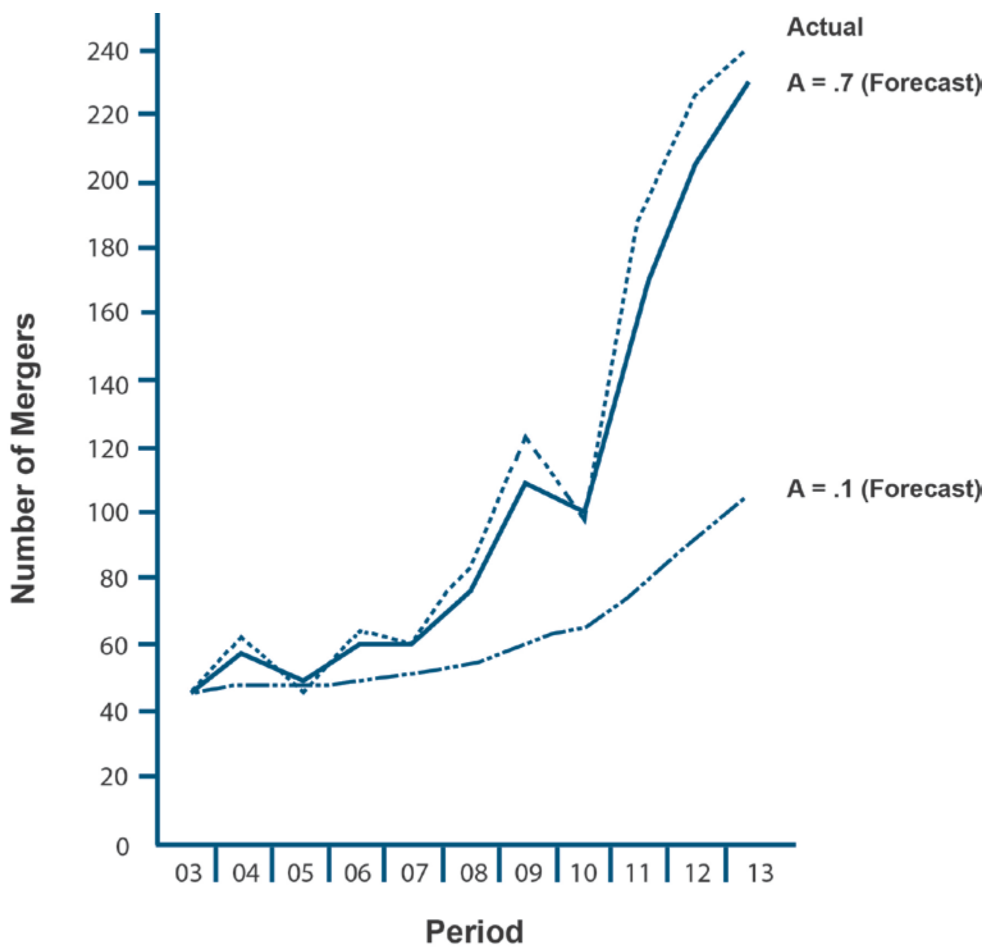
f_{01} = 46.0 mergers
f_{02} = .1(46) + .9(46.0) = 46.0
f_{03} = .1(62) + .9(46.0) = 47.6
f_{04} = .1(45) + .9(47.6) = 47.3
f_{05} = .1(64) + .9(47.3) = 49.0
f_{06} = .1(61) + .9(49.0) = 50.2
f_{07} = .1(83) + .9(50.2) = 53.5
f_{08} = .1(123) + .9(53.5) = 60.4
f_{09} = .1(97) + .9(60.4) = 64.1
f_{10} = .1(186) + .9(64.1) = 76.3
f_{11} = .1(225) + .9(76.3) = 91.2
f_{12} = .1(240) + .9(91.2) = 106.0

b. When A = .7

f_{01} = 46.0 mergers
f_{02} = .7(46) + .3(46.0) = 46.0
f_{03} = .7(62) + .3(46.0) = 57.2
f_{04} = .7(45) + .3(57.2) = 48.7
f_{05} = .7(64) + .3(48.7) = 59.4

f_{06} = .7(61) + .3(59.4) = 60.5
f_{07} = .7(83) + .3(60.5) = 76.3
f_{08} = .7(123) + .3(76.3) = 109.0
f_{09} = .7(97) + .3(109.0) = 100.6
f_{10} = .7(186) + .3(100.6) = 160.0
f_{11} = .7(225) + .3(160.4) = 205.6
f_{12} = .7(240) + .3(205.6) = 229.7

The graph shows that the high smoothing constant, A = .7, reacts more quickly to fluctuations in demand. The low value of A causes the forecast to react very slowly. Thus, the forecast is smoother.



9. a. First, convert the time variable to a simpler number. The number of appliances returned (Y) depends on time (X), the independent variable.

PERIOD	CODE(X)	APPLIANCES RETURN(Y)	(X)(Y)	X^2
5 yr,q1	1	1.2	1.2	1
q2	2	0.8	1.6	4
q3	3	0.6	1.8	9
q4	4	1.1	4.4	16

4 yr,q1	5	1.7	8.5	25
q2	6	1.2	7.2	36
q3	7	1.0	7.0	49
q4	8	1.5	12.0	64
3 yr,q1	9	3.1	27.9	81
q2	10	3.5	35.0	100
q3	11	3.5	38.5	121
q4	12	3.2	38.4	144
2 yr,q1	13	2.6	33.8	169
q2	14	2.2	30.8	196
q3	15	1.9	28.5	225
q4	16	2.5	40.0	256
1 yr,q1	17	2.9	49.3	289
q2	18	2.5	45.0	324
q3	19	2.2	41.8	361
q4	20	3.0	60.0	400
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SUM	210	42.2	512.7	2,870

$$b = \frac{n\Sigma(X)(Y) - \Sigma X\ \Sigma Y}{n\Sigma X^2 - (\Sigma X)^2}$$

$$= \frac{(20)(512.7) - (210)(42.2)}{(20)(2,870) - (210)^2}$$

$$= \frac{1,392}{13,300}$$

= 0.1047 thousands of appliances/quarter

$$a = \frac{\Sigma Y}{n} - \frac{b(\Sigma X)}{n}$$

$$= \frac{42.2}{20} - \frac{(.1047)(210)}{20}$$

= 1.011 thousands of appliances

As a result, the number of appliances returned is projected to increase 0.1047 thousand per quarter. The y-intercept is 1.011 thousand, and it represents the point on the regression line for the quarter prior to the first quarter for which data are available. The model is:

$$Y_e = 1.011 + .1047(X)$$

b. Because the periods are consecutively numbered, the second quarter of the current year is coded as 22; the projected number of appliances returned for the second quarter of the current year is:

$$Y_{22} = 1.011 + .1047(22)$$
$$= 3.314 \text{ thousand}$$

11. $f_t = A\,(x_{t-1}) + (1-A)\,f_{t-1}$

f_{Oct10} = 55.0 thousands of units
f_{Nov10} = .8(53) + .2(55.0) = 53.4
f_{Dec10} = .8(60) + .2(53.4) = 58.7
f_{Jan11} = .8(49) + .2(58.7) = 50.9
f_{Feb11} = .8(48) + .2(50.9) = 48.6
f_{Mar11} = .8(61) + .2(48.6) = 58.5
f_{Apr11} = .8(61) + .2(58.5) = 60.5
f_{May11} = .8(53) + .2(60.5) = 54.5
f_{Jun11} = .8(63) + .2(54.5) = 61.3
f_{Jul11} = .8(53) + .2(61.3) = 54.7
f_{Aug11} = .8(51) + .2(54.7) = 51.7
f_{Sep11} = .8(60) + .2(51.7) = 58.3
f_{Oct11} = .8(58) + .2(58.4) = 58.1
f_{Nov11} = .8(52) + .2(58.1) = 53.2
f_{Dec11} = .8(51) + .2(53.2) = 51.4
f_{Jan12} = .8(63) + .2(51.4) = 60.7

The forecast depends too much on the most recent months' actual demand because the smoothing constant (A) is too large. This causes the forecast to change significantly in response to any change in the prior period's actual demand.

13.

PERIOD	**CODE(X)**	**UNIT SOLD(Y)**	**(X)(Y)**	**X^2**
2010, 9	1	55	55	1
10	2	53	106	4
11	3	60	180	9
12	4	49	196	16
2011, 1	5	48	240	25
2	6	61	366	36
3	7	61	427	49
4	8	53	424	64
5	9	63	567	81
6	10	53	530	100
7	11	51	561	121
8	12	60	720	144
9	13	58	754	169
10	14	52	728	196
11	15	51	765	225
12	16	63	1,008	256
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SUM	136	891	7,627	1,496

$$b = \frac{n\Sigma(X)(Y) - (\Sigma X)(\Sigma Y)}{}$$

$$b = \frac{n\Sigma XY - (\Sigma X)(\Sigma Y)}{n\Sigma X^2 - (\Sigma X)^2}$$

$$= \frac{(16)(7,627) - (136)(891)}{(16)(1,496) - (136)^2}$$

$$= \frac{856}{5,440}$$

= 0.157 thousands of units/month

$$a = \frac{\Sigma Y}{n} - \frac{b(\Sigma X)}{n}$$

$$= \frac{891}{16} - \frac{(0.157)(136)}{16}$$

= 54.4 thousands of units

Therefore the regression line is:

$$Y_e = 54.4 + 0.157(X)$$

The periods are consecutively numbered, so Jan. 2012 is coded as 17, Feb. 2012 is coded as 18,....., June 2012 is coded as 22.

f_{Jan12} = 54.4 + 0.157(17) = 57.07 thousands of units
f_{Feb12} = 54.4 + 0.157(18) = 57.23
f_{Mar12} = 54.4 + 0.157(19) = 57.38
f_{Apr12} = 54.4 + 0.157(20) = 57.54
f_{May12} = 54.4 + 0.157(21) = 57.70
f_{Jun12} = 54.4 + 0.157(22) = 57.85

15. a.

Observation	**X**	**Y**	**(X)(Y)**	**X^2**	**Y^2**
1	154	743	114,422	23,716	552,049
2	265	830	219,950	70,225	688,900
3	540	984	531,360	291,600	968,256
4	332	801	265,932	110,224	641,601
5	551	964	531,164	303,601	929,296
6	487	955	465,085	237,169	912,025
7	305	839	255,895	93,025	703,921
8	218	478	104,204	47,524	228,484
9	144	720	103,680	20,736	518,400
10	155	782	121,210	24,025	611,524
11	242	853	206,426	58,564	727,609
12	234	878	205,452	54,756	770,884
13	343	940	322,420	117,649	883,600
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SUM	3,970	10,767	3,447,200	1,452,814	9,136,549

$$b = \frac{n\Sigma(X)(Y) - (\Sigma X)(\Sigma Y)}{n\Sigma X^2 - (\Sigma X)^2}$$

$$b = \frac{(13)(3,447,200) - (3,970)(10,767)}{(13)(1,452,814) - (3,970)^2}$$

$$= \frac{2,068,610}{3,125,682}$$

$$= .6618$$

$$a = \frac{\Sigma Y}{n} - \frac{b(\Sigma X)}{n}$$

$$a = \frac{10,767}{13} - \frac{(.662)(3,970)}{13}$$

$$= 626.1$$

$$Y_e = 626.1 + .6618(X)$$

b.

$$r = \frac{n[\sum(X)(Y)] - (\sum X)(\sum Y)}{\sqrt{[n(\sum X^2) - (\sum X)^2][n(\sum Y^2) - (\sum Y)^2]}}$$

$$r = \frac{(13)(3,447,200) - (3,970)(10,767)}{\sqrt{[(13)(1,452,814) - (3,970)^2][(13)(9,136,549) - (10,767)^2]}}$$

$$r = \frac{2,068,610}{2,983,008.8}$$

$$= 0.693$$

$$s = \sqrt{\frac{\sum Y^2 - a\sum Y - b\sum(X)(Y)}{n - 2}}$$

$$s = \sqrt{\frac{(9{,}136{,}549) - (626.1)(10{,}767) - (.6618)(3{,}447{,}200)}{11}}$$

$= 101.8$

c. The standard error of the estimate is a measure of the amount of dispersion around the regression line. The correlation coefficient measures the strength of the relationship between the variables. The higher the correlation coefficient (r) the stronger the relationship. See response to 14b.